

Long Short-Term Graph Memory Against Class-imbalanced Over-smoothing

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ABSTRACT

Most Graph Neural Networks (GNNs) follow the message-passing scheme. Residual connection is an effective strategy to tackle GNNs' over-smoothing issue and performance reduction issue on non-homophilic networks. Unfortunately, the coarse-grained residual connection still suffers from class-imbalanced over-smoothing issue, due to the fixed and linear combination of topology and attribute in node representation learning. To make the combination flexible to capture complicated relationship, this paper reveals that the residual connection needs to be node-dependent, layer-dependent, and related to both topology and attribute. To alleviate the difficulty in specifying complicated relationship, this paper presents a novel perspective on GNNs, i.e., the representations of one node in different layers can be seen as a sequence of states. From this perspective, existing residual connections are not flexible enough

for sequence modeling. Therefore, a novel node-dependent residual connection, i.e., Long Short-Term Graph Memory Network (LSTGM) is proposed to employ Long Short-Term Memory (LSTM), to model the sequence of node representation. To make the graph topology fully employed, LSTGM innovatively enhances the updated memory and three gates with graph topology. A speedup version is also proposed for effective training. Experimental evaluations on real-world datasets demonstrate their effectiveness in preventing over-smoothing issue and handling networks with heterophily.

CCS CONCEPTS

• **Computing methodologies** → **Machine learning**; • **Networks** → **Network algorithms**.

KEYWORDS

Graph Neural Networks, Long Short-Term Memory Networks, Deep Models

ACM Reference Format:

Liang Yang, Jiayi Wang, Tingting Zhang, Dongxiao He, Chuan Wang, Yuanfang Guo, Xiaochun Cao, Bingxin Niu, and Zhen Wang. 2023. Long Short-Term Graph Memory Against Class-imbalanced Over-smoothing. In *Proceedings of the 31st ACM International Conference on Multimedia (MM '23), October 29–November 3, 2023, Ottawa, ON, Canada*. ACM, New York, NY, USA, 9 pages. <https://doi.org/10.1145/3581783.3612566>

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MM '23, October 29–November 3, 2023, Ottawa, ON, Canada.

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ACM ISBN 979-8-4007-0108-5/23/10...\$15.00
<https://doi.org/10.1145/3581783.3612566>

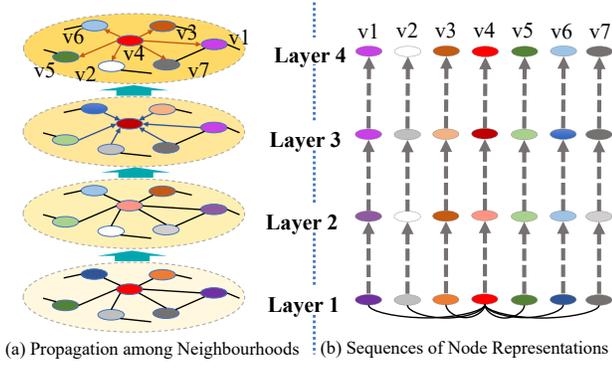


Figure 1: Different perspectives of GNNs. (a) Most existing GNNs focus on propagation among neighbourhoods. The obtained representations in previous layer are taken as the message to be propagated in next layer. (b) This paper alternatively considers the representations of one node in different layers as a sequence of states. Thus, the LSTM is employed to model these sequences.

1 INTRODUCTION

Graph Neural Networks (GNNs) [32, 41], especially Graph Convolutional Networks (GCNs), which originated from spectral graph theory, have become a kind of powerful tools for modeling irregular data in many computer vision tasks, such as sense graph generation, skeleton-based action recognition [10], subspace clustering [15] and multi-view clustering [35]. Many variants of GCNs are designed from the spatial perspective by following the message-passing scheme [7]. These models propagate messages among neighbourhoods to achieve local smoothing. However, vanilla GCNs [16] poses two serious issues, i.e., over-smoothing issue [19, 34] by stacking multiple layers and performance reduction issue [27, 42] on non-homophilic networks.

Residual connection, which is proposed to tackle vanishing gradient problem in CNNs [11], is also effective to alleviate the two issues in GCNs [4]. Different from vanilla GCN, which takes the obtained node representations in current layer as the message to be propagated in next layer, residual connection enhances the message with representations in previous layers via simple operations, such as summation and concatenation [18, 34].

Unfortunately, the coarse-grained residual connection cannot essentially solve the two issues mentioned above. Factually, the over-smoothing issue remains, and tends to be class-imbalanced. In Figure 2, the recalls of GCN with residual connection are given in class-wise with different model depths. As the model goes deeper, the recall of Class 1 is significantly higher than others. This can be attributed to that many nodes are misclassified into this class. Therefore, different nodes possess distinguishing smoothing trends, and thus should not be handled in unified strategy.

Observations mentioned above motivate us to explore a fine-grained residual connection. Recent progress explains this enhancement as seeking representations to compromise between node attribute and graph topology from the perspective of optimization [25, 37, 43]. Actually, the relationship between topology and node attribute is complicated. Therefore, to obtain robust node [23] representation, the combination of them should be flexible and nonlinear.

From the optimization perspective of GNNs [25, 37, 43], this paper reveals that the residual connection needs to be *node-dependent*, *layer-dependent*, and *related to both topology and attribute* to make the combination flexible (see Section 3 for details). Unfortunately, it is not trivial to *infer* the weights for residual connection, since it may be challenging to specify the complicated nonlinear functions between topology and attribute.

To alleviate this difficulty, this paper presents a novel perspective on GNNs, i.e., **the representations of one node in different layers can be seen as a sequence of states** as shown in Figure. 1b. Different positions on the sequence represent different hop information for the same node. Thus, the residual connection can be seen as an embedding strategy that compresses past sequential information into the current hidden state. Therefore, Long Short-Term Memory (LSTM) [12], which is proven effective in tackling vanishing and exploding gradient problems in sequence modeling, is employed to model the sequence of node representation. Benefiting from the different gates, LSTM compresses different layers of information into the current hidden state in a nonlinear and automatic way, which helps model the interaction relation among different hops.

However, the simple combination of LSTM and GNN can not make graph topology fully employed. Therefore, the first two requirements of residual connection are met but not the last. To tackle this drawback, Long Short-Term Graph Memory Network (LSTGM) is proposed by enhancing the updated memory and three gates with graph topology. By doing so, the vanilla memory state is upgraded to the graph memory state. Besides, the gate mechanism can regularize the node-level optimization process, which benefits tackling the class-imbalanced over-smoothing issue. Specifically, based on the long short-term graph memory, topology-enhanced gates can stop the training of nodes belonging to the tail class, i.e. the class with fewer nodes, while keeping the training of others.

The main contributions are summarized as follows:

- We observed the existence of class-imbalanced over-smoothing in GNN with vanilla residual connection.
- We theoretically reveal that the flexible combination of topology and attribute should be met by node- and layer-dependent residual connection.
- We propose an efficient Long Short-Term Graph Memory Network (LSTGM) to enhance updated memory and three gates in LSTM with graph topology.
- We experimentally demonstrate the superiorities of the proposed LSTGM.

2 PRELIMINARIES

This section presents the notations used in this paper, followed by the preliminaries on graph neural networks.

2.1 Notations

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote a graph with node set $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ and edge set \mathcal{E} , where N is the number of nodes. The topology of graph \mathcal{G} can be represented by its adjacency matrix $\mathbf{A} = [a_{ij}] \in \{0, 1\}^{N \times N}$, where $a_{ij} = 1$ if and only if there exists an edge $e_{ij} = (v_i, v_j)$ between nodes v_i and v_j . The degree matrix \mathbf{D} is a diagonal matrix with diagonal element $d_i = \sum_{j=1}^N a_{ij}$ as the degree of node v_i . $\mathcal{N}(v_i) = \{v_j | (v_i, v_j) \in \mathcal{E}\}$ stands for the neighbourhoods of

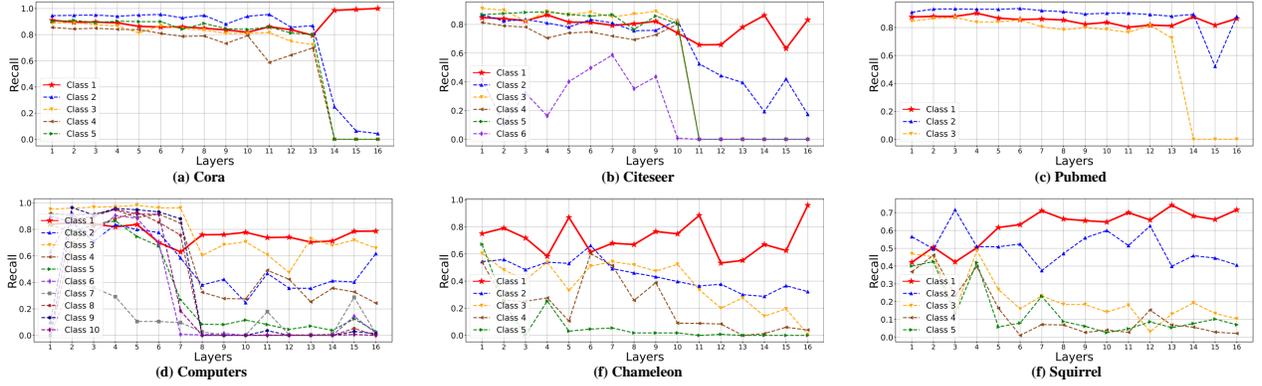


Figure 2: Class-wise recalls of GCN with residual connection as model goes deeper. For clarity, the categories are listed in descending order of the number of nodes they contain, i.e., Class 1 (in red) is the largest class.

node v_i . $\mathbf{X} \in \mathbb{R}^{N \times F}$ and $\mathbf{H} \in \mathbb{R}^{N \times F'}$ denote the collections of node attributes and representations with the i^{th} rows, i.e., $\mathbf{x}_i \in \mathbb{R}^F$ and $\mathbf{h}_i \in \mathbb{R}^{F'}$, corresponding to node v_i , where F and F' stand for the dimensions of attribute and representation.

2.2 Graph Neural Networks

Although existing graph neural networks are proposed from the perspectives of spectral and spatial, respectively, most of them follow the message passing scheme [7] based on the connection between these two perspectives [1], such as GCN [16], SGC [31]. The graph convolutional layers of GCN, SGC are as follows.

$$\text{GCN} \quad \mathbf{H}^{(t+1)} = \sigma(\tilde{\mathbf{A}}\mathbf{H}^{(t)}\mathbf{W}), \quad \mathbf{H}^{(0)} = \mathbf{X} \quad (1)$$

$$\text{SGC} \quad \mathbf{H}^{(t+1)} = \tilde{\mathbf{A}}\mathbf{H}^{(t)}, \quad \mathbf{H}^{(0)} = \mathbf{X} \quad (2)$$

where $\tilde{\mathbf{A}} = \tilde{\mathbf{D}}^{-\frac{1}{2}}\tilde{\mathbf{A}}\tilde{\mathbf{D}}^{-\frac{1}{2}}$ with $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}$ is the symmetric normalized adjacency matrix. However, these classical GNNs tend to be over-smoothing when multiple layers are employed. It makes the expressive ability of node representations and the performance significantly drop. Residual Connection is a widely-used strategy to tackle over-smoothing issue. Residual connection strategy takes the representations of previous layers as the input for the following layers, such as DeepGCNs [18] and JKNet [34]. Initial residual connection is a specific variant of residual connection, which feeds both representations from previous layer and original node attribute as the input of next layer, such as APPNP [17] and GCNII [4].

$$\text{APPNP} \quad \mathbf{H}^{(t+1)} = (1 - \alpha)\tilde{\mathbf{P}}\mathbf{H}^{(t)} + \alpha\mathbf{X} \quad (3)$$

$$\text{GCNII} \quad \mathbf{H}^{(t+1)} = \left((1 - \alpha_t)\sigma(\tilde{\mathbf{A}}\mathbf{H}^{(t)} + \alpha_t\mathbf{X}) \right. \\ \left. \times ((1 - \beta_t)\mathbf{I} + \beta_t\mathbf{W}) \right) \quad (4)$$

where $\tilde{\mathbf{P}} = \tilde{\mathbf{D}}^{-1}\tilde{\mathbf{A}}$ is the asymmetric normalized adjacency matrix. β_t is the weight for identity mapping to alleviate the overfitting issue. The weights for residual connections, i.e., α and α_t , are identical for all.

2.3 Sequential modeling

Recurrent models such as LSTM are not the first time introduced to graph modelling. There are mainly two kinds of strategies to combine LSTM and GNN. The first strategy modifies the network

topology to control the aggregation. GraphSAGE [9] samples and reorders neighbour nodes, then follows an LSTM unit as information aggregation. The second strategy keeps intermediate node embedding and uses LSTM as a pooling method. JKNet [34] employs bidirectional LSTM to learn layer-wise attention score, while Geniepath [22] propagates the outputs of LSTM and feeds them to the next LSTM unit. The second strategy, more similar to the proposed LSTMG, can be formulated as LSTM-regularized GNNs. Furthermore, this kind of GNN is shown in Figure. 3(c). However, the backbone GNN and the employed LSTM are two individual components, and the graph topology can not be fully employed by the LSTM.

3 OBSERVATIONS AND ANALYSIS

Observation: In Figure 2, the recalls of GCN with residual connection are given in class-wise with different model depths. For clarity, the categories are listed in descending order of the number of nodes they contain, i.e., Class 1 is the largest class. As the model goes deeper, the recall of Class 1 is significantly higher than others. At the same time, the recalls of other classes remarkably drop. The more significant the difference in the number of nodes of each category is, the more pronounced the class-imbalanced over-smoothing is. This can be attributed to that many nodes from other classes are misclassified into Class 1. This indicates that nodes in different classes possess different smoothing trends. Therefore, different nodes possess distinguishing over-smoothing trends, and thus should not be handled in unified strategy, when their categories are unknown.

Residual connections are widely known to alleviate the over-smoothing issue slightly. However, when the superficial problem is deepened into a deep-seated one, i.e. from over-smoothing to class-imbalanced over-smoothing, it is necessary to redesign the residual connection strategy. Figure 2 reminds us that class-imbalanced phenomenon is universal in homophily and heterophily networks. Thus a good residual connection strategy should tackle the class-imbalanced over-smoothing problems and perform well both in homophily and heterophily networks. Then the direction to explore is what properties such a residual connection should be met.

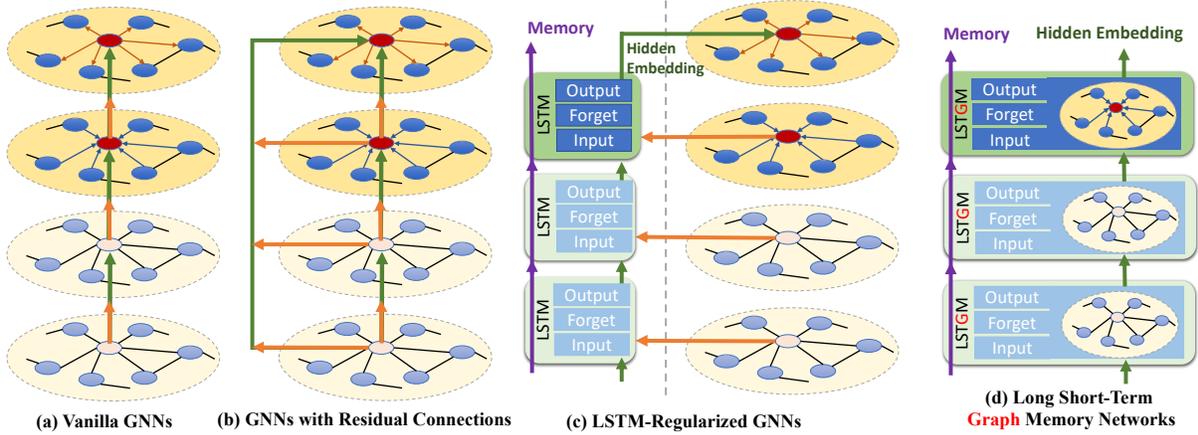


Figure 3: Comparisons between Vanilla GNNs, GNNs with residual connections, the proposed LSTM-regularized GNNs and Long Short-Term Graph Memory Networks. (a) Vanilla GNNs directly propagate the obtained node representations in last layer. (b) GNNs with residual connections combine representations in previous layers as message for all nodes in a uniform manner. (c) LSTM-regularized GNNs specify messages for different nodes from their representations in previous layers via LSTM. LSTM-regularized GNNs are equivalent to only employing topology information in the output gate in the LSTM. (d) Long Short-Term Graph Memory Networks (LSTGM) enhance LSTM-regularized GNNs by integrating topology information into input gate, forget gate and output gate. Refer to Figure. 4 for the detail.

Analysis: Recently, AUC optimization has achieved great success on long-tailed classification [38] and obtain state-of-the-art performance on complicated tasks such as adversarial training [13] and performance-constrained optimization [39]. Moreover, [3] present a first-trial to introduce topology-aware AUC optimization on Graph. Besides, some works interpret and unify graph convolutional networks from the perspective of the numeral optimization [25, 37, 43]. Specifically, they show that the graph convolution with residual connection, i.e. Eq. (3), is to minimize the following objective function via gradient descent

$$\begin{aligned} C &= \|\mathbf{X} - \mathbf{H}\|_F^2 + \lambda \text{tr}(\mathbf{H}^T \tilde{\mathbf{L}} \mathbf{H}) \\ &= \sum_{i=1}^N \|\mathbf{x}_i - \mathbf{h}_i\|_2^2 + \frac{\lambda}{2} \sum_{i=1}^N \sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i - \mathbf{h}_j\|_2^2, \end{aligned} \quad (5)$$

where \tilde{a}_{ij} represents the element of matrix $\tilde{\mathbf{A}}$, while $\tilde{\mathbf{L}} = \mathbf{I} - \tilde{\mathbf{A}}$ represents the Laplacian matrix of adjacency matrix $\tilde{\mathbf{A}}$. The first term stands for the distance between the node embeddings \mathbf{h}_i 's and attributes \mathbf{x}_i 's, while the second term stands for the distance between the representations of two connected nodes \mathbf{h}_i and \mathbf{h}_j . The gradient of C with respected to \mathbf{h}_i is

$$\begin{aligned} \frac{\partial C}{\partial \mathbf{h}_i} &= \frac{\partial \|\mathbf{x}_i - \mathbf{h}_i\|_2^2}{\partial \mathbf{h}_i} + \frac{\lambda}{2} \frac{\partial \sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i - \mathbf{h}_j\|_2^2}{\partial \mathbf{h}_i} \\ &= (\mathbf{x}_i - \mathbf{h}_i) + \lambda \left(\sum_{j \in N(i)} \tilde{a}_{ij} (\mathbf{h}_i - \mathbf{h}_j) \right). \end{aligned} \quad (6)$$

By setting the gradient as zero, the updating rule can be obtained as

$$\mathbf{h}_i^{(t+1)} = \alpha \sum_{j \in N(i)} \tilde{a}_{ij} \mathbf{h}_j^{(t)} + \beta \mathbf{x}_i, \quad (7)$$

where α and β are the node-independent parameters to balance the impacts from its attribute \mathbf{x}_i and representations of its neighbourhoods. By comparing Eq. (7) with Eq. (3), graph convolutional operation can be seen as the gradient descent of objective function Eq. (5), and $\beta \mathbf{x}_i$ in Eq. (7) can be regarded as the initial residual connection.

However, one remarkable drawback of the objective function in Eq. (5) is the *linear combination* of the two terms. Therefore, the widely-used graph convolutional operation also inherits this drawback. Actually, the combination of the impacts from topology and attribute may be complicated. To this end, Eq. (5) can be generalized to

$$\mathcal{F} = \sum_{i=1}^N f(\|\mathbf{x}_i - \mathbf{h}_i\|_2^2) + \frac{\lambda}{2} \sum_{i=1}^N g\left(\sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i - \mathbf{h}_j\|_2^2\right), \quad (8)$$

where $f(\cdot)$ and $g(\cdot)$ represent the nonlinear function to combine the impacts from attribute and topology. The gradient of \mathcal{F} with respected to \mathbf{h}_i is

$$\begin{aligned} \frac{\partial f(t_i)}{\partial t_i} \frac{\partial \|\mathbf{x}_i - \mathbf{h}_i\|_2^2}{\partial \mathbf{h}_i} + \frac{\partial g(s_i)}{\partial s_i} \frac{\lambda}{2} \frac{\partial \sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i - \mathbf{h}_j\|_2^2}{\partial \mathbf{h}_i} \\ = \gamma_i (\mathbf{x}_i - \mathbf{h}_i) + \lambda \eta_i \left(\sum_{j \in N(i)} \tilde{a}_{ij} (\mathbf{h}_i - \mathbf{h}_j) \right), \end{aligned}$$

where $t_i = \|\mathbf{x}_i - \mathbf{h}_i\|_2^2$ represents the distance between node representation and attribute, while $s_i = \sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i - \mathbf{h}_j\|_2^2$ stands for the distance of representations between nodes and its neighbourhoods. By comparing with Eq. (6), $\gamma_i = \frac{\partial f(t_i)}{\partial t_i}$ and $\eta_i = \frac{\partial g(s_i)}{\partial s_i}$ are node-dependent weights, which balance the impacts from two terms. Taking $f(x) = g(x) = \sqrt{x}$ as an example, $\gamma_i = \frac{1}{\sqrt{t_i}}$ and $\eta_i = \frac{1}{\sqrt{s_i}}$ stand for the reconstruction errors of attribute and topology from the representation of node v_i , respectively. Similar to

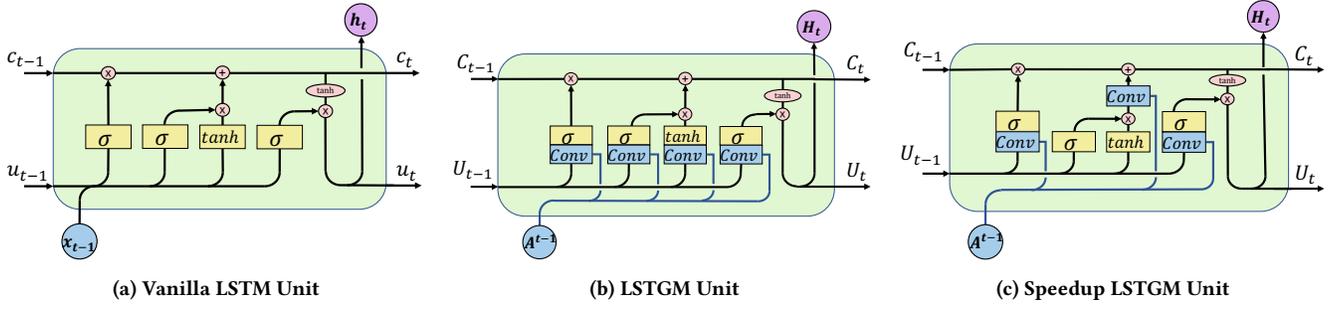


Figure 4: From LSTM to the proposed Long Short-Term Graph Memory Networks (LSTGM). (a) The vanilla LSTM. (b) The proposed Long Short-Term Graph Memory Networks (LSTGM). LSTGM enhances input, forget and output gates with graph topology information. (c) The LSTGM speedup. Two convolutional operations are combined.

updating rule in Eq. (7), the updating rule for Eq. (8) is

$$\mathbf{h}_i^{(t+1)} = \alpha_i^{(t)} \sum_{j \in N(i)} \tilde{a}_{ij} \mathbf{h}_j^{(t)} + \beta_i^{(t)} \mathbf{x}_i, \quad (9)$$

where $\alpha_i^{(t)}$ and $\beta_i^{(t)}$ are node- and iteration-dependent weights, which is related to reconstruction errors of $t_i^{(t)} = \|\mathbf{x}_i - \mathbf{h}_i^{(t)}\|_2^2$ and $s_i^{(t)} = \sum_{j \in N(i)} \tilde{a}_{ij} \|\mathbf{h}_i^{(t)} - \mathbf{h}_j^{(t)}\|_2^2$. Note that the iteration in updating rule (Eq. (9)) is the layer in GNNs (Eqs. (1)-(4)). Especially, the weight $\beta_i^{(t)}$ for residual connection should possess three demanded properties.

- The weights should be node-dependent instead of identity for all nodes.
- The weights should be layer-dependent, since it is the function of representations in the previous layers.
- The weights should be related to both graph topology and node attributes.

Unfortunately, it is not trivial to *infer* the weights $\alpha_i^{(t)}$ and $\beta_i^{(t)}$ as shown above, since it may be difficult to specify the complicated nonlinear functions $f(\cdot)$ and $g(\cdot)$. To alleviate this issue, this paper tends to *learn* them via a highly expressive and robust model in next section.

4 METHODOLOGY

As discussed in previous section, flexible weights for residual connection may benefit modeling the complicated relationship between topology and attribute on node representation learning. To this end, this section combines Long Short-Term Memory (LSTM) in GNNs to learn the flexible residual connection.

4.1 Long Short-Term Graph Memory Network

As shown in Figure. 3(b), vanilla residual connection often combines representations from previous layers for propagation in next layer in a uniform manner, such as summation and concatenation. Note that **the representations of one node in different layers, i.e., $\{\mathbf{h}_i^{(t)}\}_{t=1}^T$, can be seen as a sequence of states**. From this perspective, vanilla residual connection can be regarded as a simple sequence model, which predicts next state with previous states via a fixed simple strategy. Unfortunately, existing simple strategies, such as summation and concatenation, are not flexible and robust [24] for sequence modeling. Thus, this section employs LSTM to

model the sequence of node representation. LSTM is proved to be effective in tackling vanishing and exploding gradient problems in long range dependence modeling.

However, simple combination of LSTM and GNN is imperfect. As shown in Figure.3(c), the backbone GNN and the employed LSTM are two individual components, which are bridged by the node representations. This causes the specific residual connections to be only based on the node representations in previous layers, and the graph topology can not be fully employed by the LSTM. Therefore, the first two requirements for the residual connection are met but not the third.

To alleviate this issue, Long Short-Term Graph Memory Network (LSTGM) is proposed by enhancing the updated memory and three gates with graph topology as shown in Figure.3(d). The vanilla LSTM unit shown in Figure. 4a consists of a memory component $\mathbf{c}^{(t)}$ and a hidden component $\mathbf{u}^{(t)}$. Then the architecture of LSTGM unit is shown in Figure. 4b. Different from LSTM, which models i.i.d. sequences, the proposed LSTGM tends to model the sequence of graph data. To this end, LSTGM takes adjacency matrices from different orders, i.e., \mathbf{A}^t , as input. Thus, the three gates and updated memory can be enhanced with these adjacency matrices as

$$\mathbf{F}^{(t)} = \sigma(\mathbf{A}^t \mathbf{U}^{(t-1)} \mathbf{W}_f), \quad (10)$$

$$\mathbf{I}^{(t)} = \sigma(\mathbf{A}^t \mathbf{U}^{(t-1)} \mathbf{W}_i), \quad (11)$$

$$\mathbf{O}^{(t)} = \sigma(\mathbf{A}^t \mathbf{U}^{(t-1)} \mathbf{W}_o), \quad (12)$$

$$\tilde{\mathbf{C}}^{(t)} = \tanh(\mathbf{A}^t \mathbf{U}^{(t-1)} \mathbf{W}_c), \quad (13)$$

where $\mathbf{F}^{(t)}$, $\mathbf{I}^{(t)}$, $\mathbf{O}^{(t)}$ are the three gates for all nodes, while $\tilde{\mathbf{C}}^{(t)}$ is the updated memory for all nodes. Based on this, the memory and hidden state can be formulated as

$$\mathbf{C}^{(t)} = \mathbf{F}^{(t)} \otimes \mathbf{C}^{(t-1)} + \mathbf{I}^{(t)} \otimes \tilde{\mathbf{C}}^{(t)}, \quad (14)$$

$$\mathbf{U}^{(t)} = \mathbf{O}^{(t)} \otimes \mathbf{C}^{(t)}, \quad (15)$$

where \otimes denotes element-wise product. LSTGM needs four graph convolution operations and some of them are redundancy. Therefore, we give a speedup version for efficient.

Speedup: To update the memory, both $\mathbf{I}^{(t)}$ and $\tilde{\mathbf{C}}^{(t)}$ needs per-form graph convolution. Therefore, by exchanging the order of nonlinear mapping and graph convolution, Eqs. (11), (13) and (14)

Table 1: Datasets statistics

Dataset	Cora	Citeseer	Pubmed	Computers	Photo	Chameleon	Squirrel	Actor	Texas	Cornell
# Nodes	2,708	3,327	19,717	13,752	7650	2,277	5,201	7,600	183	183
# Edges	5,429	4,732	44,338	245,861	119,081	36,101	217,073	33,544	309	295
# Features	1,433	3,703	500	767	745	2,325	2,089	931	1,703	1,703
# Classes	7	6	3	10	8	5	5	5	5	5
Homphily Rate	0.83	0.71	0.79	0.79	0.84	0.25	0.22	0.24	0.06	0.11

Table 2: Large-scale graph statistics

Dataset	Ogbn-products	Ogbn-mag
# Nodes	2,449,029	1,939,743
# Edges	61,859,140	21,111,007
# Features	100	128
# Classes	47	349
# Train/Val/Test	10%/2%/88%	85%/9%/6%

can be reformulated as

$$\mathbf{I}^{(t)} = \sigma(\mathbf{U}^{(t-1)}\mathbf{W}_i), \quad (16)$$

$$\tilde{\mathbf{C}}^{(t)} = \tanh(\mathbf{U}^{(t-1)}\mathbf{W}_c), \quad (17)$$

$$\mathbf{C}^{(t)} = \mathbf{F}^{(t)} \otimes \mathbf{C}^{(t-1)} + \mathbf{A}^t(\mathbf{I}^{(t)} \otimes \tilde{\mathbf{C}}^{(t)}). \quad (18)$$

The architecture for speedup LSTGM is shown in Figure. 4c. It only needs three graph convolution operations. In practice, the first and third convolution operation from left to right have the same input, hence the convolution operation can be further reduced to two. Furthermore, the feature transformation in convolution can be removed for simplicity.

Remark: The proposed LSTGM is very different from Spatial Temporal Graph Neural Networks (STGNNs), such as Graph-WaveNet [33], CGCN [40], ST-GCN [36], TSSRGCN [5] and ASTGCN [8] from both solved problem and data structure. LSTGM focuses on static graph and tends to alleviate the weakness in residual connection, while STGNNs focus on dynamic graph and model spatial temporal changes. To the best of our knowledge, LSTGM is the first to model representation in layers via sequence model.

5 EVALUATIONS

5.1 Experimental Setup

Real-world Datasets. The proposed LSTGM is validated on 10 networks, whose statistics are shown in Table 1. These 10 networks can be categorized into 4 categories. **Citation networks:** Cora, Citeseer, and Pubmed are the standard citation network benchmark datasets [26, 29]. **WebKB webpage networks:** Cornell and Texas are the webpage networks. **Co-occurrence network:** Actor network contains the co-occurrences of actors in films. **Wikipedia networks:** Chameleon and Squirrel are the webpages extracted from different topics in Wikipedia [28]. In Table 1, the network homophily rates [27] are provided. **Large-scale graph.** Ogbn-products and Ogbn-mag [14]. The statistics about these 2 datasets are summarized in Table 2.

Baseline Methods. To verify the effectiveness LSTGM, 13 baseline methods are employed. They are divided into 3 categories: **Classic GNNs** for node classification task include vanilla GCN [16], GAT [30], and GraphSAGE [9]. **GNNs with residual connection** include JKNet [34], APPNP [17], and GCNII [4]. **GNNs for networks with**

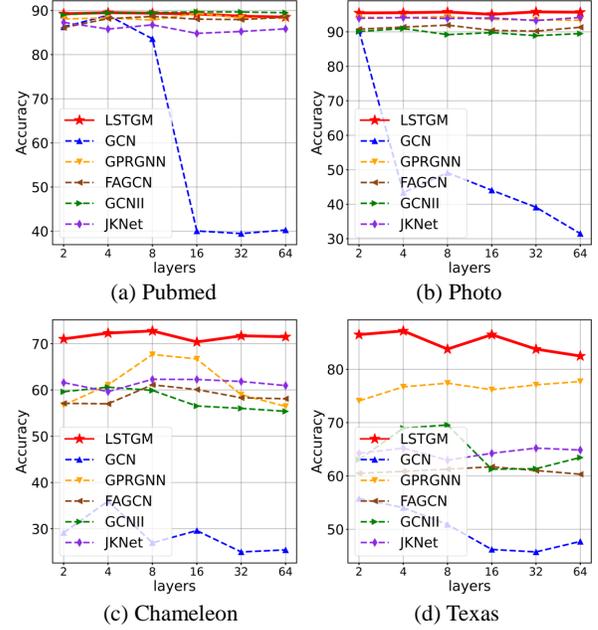


Figure 5: Node classification results with various model depths.

heterophily include Geom-GCN [27], H2GCN [42], GPRGNN [6], and FAGCN [2]. **GNNs for large-scale graphs** include LINKX [20] and GloGNN [21].

Parameter Setting. For datasets in Table. 1, we randomly split nodes of each class into 60%, 20% and 20% for training, validation and testing. All results are obtained by computing over 10 random splits, as suggested in [27]. The hyperparameters, including weight decay, dropout, and learning rate, are tuned on validation set.

5.2 Results Analysis

5.2.1 Results on Real-World Datasets. The results are shown in Table 3, where the bold and the underlined indicate the best and the second best performances, respectively.

On small homophilic datasets, i.e., Cora, Citeseer, and Pubmed, LSTGM achieves comparable performances, which are slightly lower than SOTA, and the differences are very tiny. It can be observed that LSTGM achieves new remarkable SOTA results on large homophilic datasets Computer and Photo, compared to all other baselines. It demonstrates that the node-dependent weights for residual connections are indispensable.

On heterophilic datasets, LSTGM is compared with Geom-GCN, GPRGNN, FAGCN, and H2GCN, which are all the GNNs designed

Table 3: Classification Accuracy (Bold indicates the best, underlined indicates the second best).

Methods	Cora	Citeseer	Pubmed	Computer	Photo	Chameleon	Squirrel	Actor	Texas	Cornell
GCN	85.77±0.25	73.68±0.31	88.13±0.28	82.52±0.32	90.54±0.21	35.99±2.58	34.02±1.34	26.97±1.49	55.68±9.61	55.14±7.57
GAT	86.37±0.30	74.32±0.27	87.62±0.26	81.95±0.38	90.09±0.27	60.26±2.50	40.72±1.55	27.44±0.89	58.38±4.45	58.92±3.32
GraphSAGE	87.77±1.04	71.09±1.30	88.42±0.50	83.11±0.23	90.51±0.25	58.73±1.68	41.61±0.74	34.23±0.99	82.43±6.14	75.95±5.01
MLP	74.82±2.22	70.94±0.39	63.76±0.78	70.48±0.28	78.69±0.30	46.21±2.99	28.77±1.56	36.53±0.70	81.89±4.78	81.08±6.37
APPNP	87.87±0.85	76.53±1.33	89.40±0.61	81.99±0.26	91.11±0.26	54.30±0.34	33.29±1.72	31.71±0.70	82.43±1.72	82.16±3.83
GCNII	<u>88.49±2.78</u>	<u>77.08±1.21</u>	89.57±1.56	86.13±0.51	90.98±0.93	60.61±2.00	37.85±2.76	36.18±0.61	69.46±1.86	74.86±2.73
JKNet	88.93±1.35	74.37±1.53	87.68±0.30	77.80±0.97	94.13±0.70	62.31±2.76	44.24±2.11	36.47±0.51	65.35±4.86	56.49±3.22
DAGNN	87.40±0.72	74.67±1.31	84.84±0.35	88.35±0.24	94.36±0.25	53.79±1.29	37.68±0.63	30.50±0.59	79.19±2.42	74.32±3.47
Geom-GCN-I	85.19±1.13	77.99±1.23	90.05±0.90	NA	NA	60.31±1.77	33.32±1.59	29.09±0.86	57.58±1.97	56.76±3.17
Geom-GCN-P	84.93±0.51	75.14±1.50	88.09±1.37	NA	NA	60.90±1.13	38.14±1.23	31.63±0.98	67.57±1.13	60.81±2.21
GPRGNN	88.65±1.37	77.99±1.64	89.18±0.61	89.43±0.86	94.76±0.20	67.48±1.98	49.93±1.34	36.58±1.04	77.84±2.78	79.73±3.91
FAGCN	87.77±1.69	74.66±2.27	88.60±0.64	86.09±0.40	91.96±0.71	61.12±1.95	40.88±2.02	<u>36.81±0.26</u>	61.82±8.71	67.95±10.02
H2GCN	86.92±1.37	76.88±1.77	89.40±0.34	86.67±0.32	93.91±0.48	59.39±1.98	37.90±2.02	<u>35.62±1.30</u>	<u>84.86±4.32</u>	<u>82.16±3.27</u>
LSTM-GNNs	83.98±1.15	73.94±1.24	88.78±0.23	<u>89.90±0.81</u>	<u>94.81±0.17</u>	<u>69.66±0.49</u>	<u>56.55±1.65</u>	36.51±0.65	80.54±5.65	81.62±4.09
LSTGM	86.72±0.55	76.41±0.90	<u>89.93±0.16</u>	90.85±0.32	95.68±0.29	73.77±1.01	61.36±0.91	37.26±0.57	85.41±4.18	84.59 ± 4.17

Table 4: Classification Accuracy on Large-scale Graph(Bold indicates the best, underlined indicates the second best).

Dataset	Ogbn-products		Ogbn-mag	
	Val Accuracy	Test Accuracy	Val Accuracy	Test Accuracy
GCN	92.00±0.03	75.64±0.21	39.66±0.18	39.02±0.16
GraphSAGE	92.24±0.07	80.50±0.14	43.68±0.06	42.28±0.21
APPNP	91.84±0.07	79.71±0.50	42.81±0.15	42.50±0.45
GPRGNN	93.02±0.13	81.95±0.14	50.24±0.30	49.48±0.34
LINKX	93.31±0.03	<u>83.30±0.25</u>	53.62±0.06	53.14±0.19
GloGNN	93.42±0.14	83.28±0.51	54.77±0.15	<u>53.86±0.13</u>
LSTGM	93.05±0.02	84.42±0.22	54.84±0.62	55.11±0.27

for handling networks with heterophily. This demonstrates that topology and node attributes-related adaptive residual is critical in heterophilic networks on account of the complicated topology information and poor predictability attributes are widespread in networks with heterophily. In addition, by sequence modeling, layer-dependent residual connections benefit LSTGM by capturing different order information.

LSTGM can handle large-scale graphs. As shown in Table 4, LSTGM achieves the best results on Ogbn-products and Ogbn-mag. These results inspire a broader industrial application for LSTGM.

5.3 Preventing Over-smoothing Issue

To prove that the proposed LSTGM can alleviate over-smoothing issue, five baselines such as GCN, GCNII, FAGCN, GPRGNN and JKNet, which perform well as deep models, are compared with the proposed LSTGM with varying numbers of layers on 8 networks mentioned in Table 1. The results are shown in Figure 5.

As shown in Figure 5(a)-(b), the proposed LSTGM keeps the performance stable or continues to rise with the number of layers increasing. This phenomenon is mainly attributed to the adaptive residual connections, which enable fine-grained consideration. For different nodes and layers, the residual information regularized by

Table 5: Model Efficiency: average total running time (s)

Model	GCNII	GPRGNN	H2GCN	LSTGM
Pubmed	31.81	22.39	46.29	65.18
Computer	18.15	12.52	33.50	40.84
Actor	11.35	10.36	20.97	29.17
Squirrel	9.11	6.17	15.39	21.06

the LSTGM is different, ensuring that redundant information will not be propagated frequently, thus alleviating the over-smoothing. As shown in Figure 5(c)-(d), the proposed LSTGM outperforms other deep models, which are specially designed for heterophilic networks. This phenomenon proves that a more comprehensive and appropriate residual design is vital for heterophilic networks.

Compared to the case of GCN with residual connection in Figure 2, Figure 6 provides the class-wise recalls of LSTGM as model goes deeper. It can be observed that LSTGM overcome the class-imbalanced over-smoothing issue, due to its node- and layer-dependent fine-grained residual connection in LSTGM.

5.4 Efficiency study

In this section, we study efficiency. We compare the average training time for effective methods on relatively large datasets for fairness. We use the same training set for all these methods on each dataset and run the experiments for 500 epochs.

As shown in 5, though the average total training time of LSTGM is larger than others, the gap is minimal. GPRGNN get the fewest running time, but the classification accuracy on heterophily networks is far behind LSTGM as shown in 3. GCNII and H2GCN introduce new trainable parameters for every layer, while LSTGM's parameters are agnostic of layers and reduce the time cost consequently. In general, the additional time cost is acceptable and LSTGM is efficient.

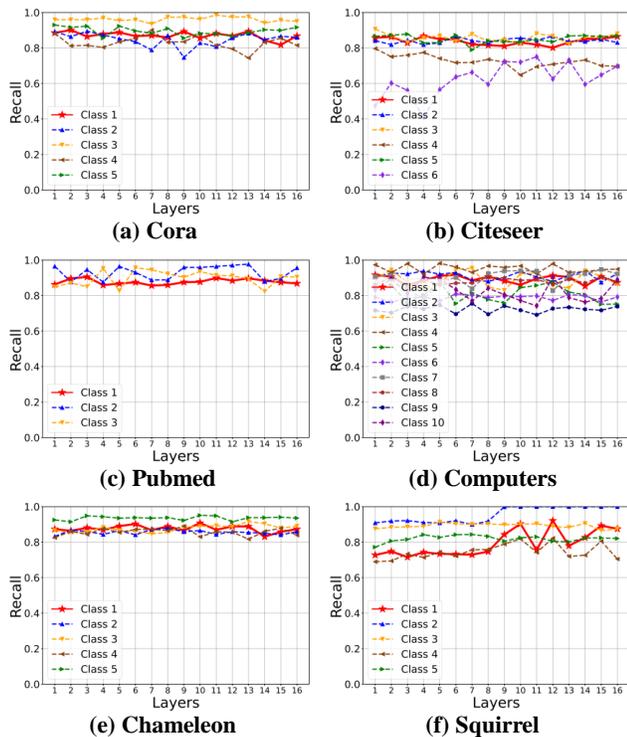


Figure 6: Class-wise recalls of LSTGM as model goes deeper.

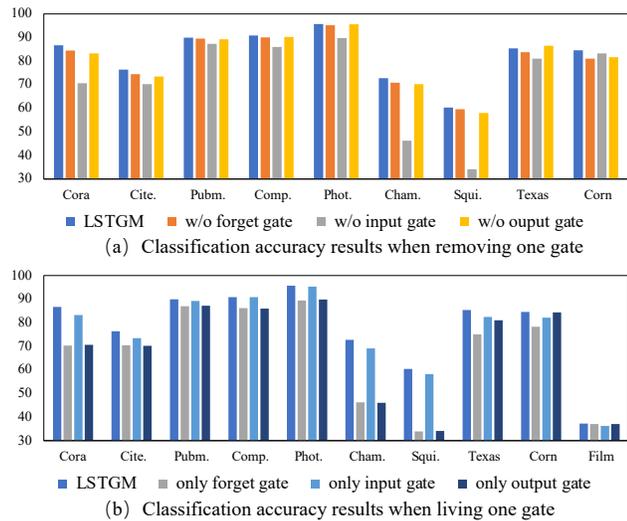


Figure 7: The accuracies of LSTGM with different parts.

5.5 Ablation Study

This section performs the ablation study. In the first experiment, one of the three gates is kept. As shown in Figure 7(a), dramatic performance degradation can be observed once we remove the input gate in most datasets. Forget and output gate contribute more to the long-term dependencies, which tell nodes what they should be forgotten and should be passed. In the second experiment, one of three gates is removed respectively. Figure 7(b) also intuitively shows the importance of different gates on various datasets. They come to a similar conclusion as Figure 7(a).

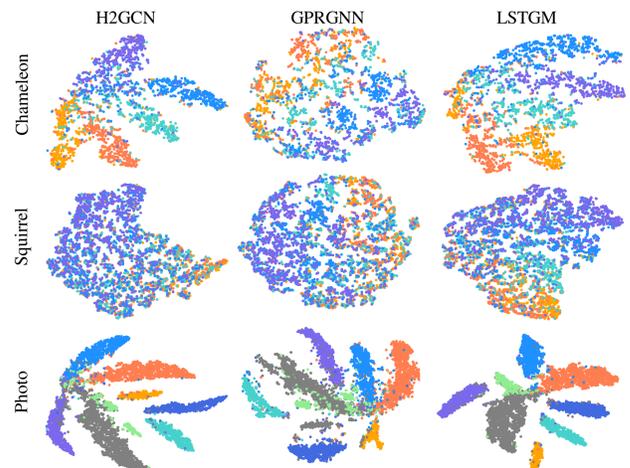


Figure 8: t-SNE of the node representations.

5.6 Visualizations

To provide an intuitive understanding of LSTGM, the t-SNE visualizations of the node embeddings obtained from different GNNs, including GPRGNN, H2GCN, and the proposed LSTGM on the Chameleon, Squirrel, and Photo, are shown in Figure 8. The colors of nodes represent their labels. The proposed LSTGM occurs relatively little overlapping phenomenon on heterophilic datasets. Besides, the representations on homophilic networks are more discriminative than those from other methods. Therefore, the expressive power of LSTM-GNN is high.

6 CONCLUSIONS

The coarse-grained node-independent residual connection still suffers from class-imbalanced over-smoothing issue, due to the fixed and linear combination of topology and attribute in node representation learning. To make the combination flexible to capture complicated relationship, this paper reveals that the residual connection needs to be node-dependent, layer-dependent, and related to both topology and attribute, and implements these requirements via LSTM by considering the representations of one node in different layers as a sequence of states. Furthermore, Long Short-Term Graph Memory Network (LSTGM) is proposed by updating memory to graph memory and enhancing the three gates with graph topology. Experimental evaluations demonstrate that LSTGM possesses the attractive characteristics including overcome the class-imbalanced over-smoothing and superior performance on real-world networks including homophilic, heterophilic and large networks.

7 ACKNOWLEDGMENTS

This work was supported in part by the National Science Fund for Distinguished Young Scholarship of China (No. 62025602), in part by the National Natural Science Foundation of China (No. 61972442, 62102413, 62276187, 62272020, U1936210, U1936208, U22B2036, 11931915), in part by the Natural Science Foundation of Hebei Province of China under Grant F2020202040, in part by the Fok Ying-Tong Education Foundation China (No. 171105), the Tencent Foundation and XPLOER PRIZE, and in part by the Fundamental Research Funds for the Central Universities.

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